Abstract

This paper argues that testing participation – and not testing capacity – is the biggest obstacle to a successful “test and isolate”-strategy, as recently proposed by Paul Romer. If $R_0 = 2.5$, at least 60 percent of a population needs to participate in a testing program to make it theoretically possible to achieve an effective reproduction rate for the whole population, $R''$, below 1. I also argue that Paul Romer’s assumption about quarantine length is problematic, because it implicitly assumes that an infected and tested person is quarantined during the entire duration of the illness. With more realistic assumptions, where the fraction of the illness duration that is spent in quarantine depends on the test frequency, at least 80 percent of the population must participate to keep $R''_0 < 1$, even if participants in the test program are tested every five days. Comprehensive testing, as proposed by Romer, is probably still a very cost-effective means of reducing the reproduction rate of the infection compared to mandatory lockdown policies, but it seems less promising than he suggests. However, comprehensive testing might also reduce voluntary social distancing in a non-cost-effective way because testing and isolating infected individuals decreases the risk of infection for an individual if social distancing is not practiced.

Introduction and Summary

Governments around the World are looking for more efficient ways to dance with COVID-19 than broad lockdowns. In a widely cited webinar presentation given on April 3, Paul Romer proposed population-wide testing and isolation (Paul M. Romer 2020). Romer made several important points. First, the economic benefit of a speedier recovery is in the order of trillions of dollars and can easily justify spending billions on a test program. Second, the quality of the tests is not a barrier for a successful “test and isolate”-strategy, since a high rate of false negatives (Romer uses 30 percent false negatives) can be compensated for by more testing. Third, the market is likely to be able to deliver testing capacity very quickly, if regulations and red tape (which are focused on clinical situations where test precision is very important) are removed.

Based on a simple model, Romer shows that, if we test just 7 percent of the population each day, we can keep the effective reproduction rate, $R'$ (the number of person an infected person infects) below 1, thereby keeping COVID-19 under control.

Unfortunately, as I will show, Romer’s model has some important flaws, and more importantly, participation in the test program seems to be a bigger problem for a successful “test and isolate”-

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1 “Dancing with COVID-19” is a term adopted from the influential blogpost “Coronavirus: The Hammer and the Dance”, see (Pueyo 2020)
strategy than testing capacity (here I agree with Paul Romer, and think that the market will be able to deliver sufficient capacity if the incentives (i.e. profits) are strong enough).

Romer’s idea and model

The reproduction rate $R$ in an epidemic is the expected number of cases directly generated by any one infectious case. The basic reproduction rate, $R_0$, is the initial value of $R$ when all individuals are susceptible to infection, and no suppression policies have been applied.

An epidemic can only be halted if the value of $R$ is brought below 1. When that happens, each infected person infects less than one new person, and the epidemic will die out. If that does not happen, more and more people become infected until – ultimately – “herd immunity” is achieved.

Romer wants to reduce the effective reproduction rate $R'$ to $R' = 0.75$ from $R' = 2.5$, by randomly testing a fraction of the entire population each day and then isolating those who are found to be positive. In Romer’s analysis, which we follow, the effective reproduction rate $R'$ is the product of the basic reproductive rate, $R_0$, and the fraction of the infectious population that is not isolated. $R'$ is below $R_0$ since a fraction of the population is tested each day and those found to be infectious are isolated. Let $\varphi$ be the proportion of the infectious population who are isolated. Then, Romer writes equation (1):

$$ R' = (1 - \varphi) \cdot R_0 $$

(1)

where $\varphi$ is given by equation (2):

$$ \varphi = t \cdot (1 - n) \cdot l $$

(2)

where $t =$ fraction of infected tested daily (equal to fraction of population in a random sample), $n =$ share of false negatives, and $l =$ length of isolation. As noted by (Cleavelly et al. 2020) the interpretation of equation (2) is somewhat problematic, but for now it suffices to interpret $\varphi$ as the long-run share of the population which is quarantined. I will get back to the problems with equation (2) later.

For $R' = 0.75$, equation (1) implies that that $\varphi = 0.7$, i.e. that 70 percent of the infectious population is isolated, and that sufficient tests and isolation are carried out to make this possible. Drawing on his calculations, Romer suggests that this can be done by randomly testing 7 percent of the population each day. He believes that this would achieve $R' = 0.75$. With a population of 300 million in the US, testing on this scale would require about 20 million tests a day. In Denmark, with a population of 5.8 million, this would require about 400,000 tests a day.

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2 There are SEIR-models where the epidemic can spread even when $R<1$ because of the existence of super spreaders. See (Reich, Shalev, and Kalvari 2020) for an interesting case of this.
Romer implicitly assumes that everybody (100 percent of the population) will participate in the test program and does not discuss how this could be achieved. As I will show, participation is a major obstacle to Romer’s idea and a “test and isolate”-strategy is unlikely to bring $R’$ below 1.

Below, I will first demonstrate how participation matters in Romer’s model, where it is implicitly assumed that an infected person is tested on the first day of his/her illness. I will then extend Romer’s model to a more realistic model, where the fraction of the illness duration spent in quarantine depends on the test frequency, and see how participation rates affect the outcomes of a “test and isolate”-strategy.

I will show that, if the participation rate is below 60 percent, it is theoretically impossible to achieve $R’ < 1$. And even if 100 percent participate, we need to test almost 2½ times as many people every day as Romer suggests, when the fraction of the illness period spent in quarantine depends on the test frequency.

**Participation rates matters greatly**

Romer implicitly assumes that everybody will participate in a “test and isolate”-strategy, but this assumption is far from realistic. Participating in a test program has several adverse personal consequences, giving people incentives not to participate or to shirk/cheat if participating.

First, there are limited personal benefits from participation. If you are asymptotic, there is no benefit from knowing you are infected, unless you are in contact with people in the risk group (which will primarily be the obese, elderly and chronically ill) that you personally want to protect. Second, participation is not free. Even if the test itself is provided by the government, the participant still needs to spend time on the test and fit the testing into daily routines. Third, testing positive can have serious consequences for the individual and her/his network, if they must isolate despite having no symptoms.

Hence, there are reasons to believe that only a fraction of the population will participate in a test program. We can calculate the effective reproduction rate which depends on the share of the population who participate, as stated in equation (3):

$$R'' = \alpha \cdot R' + (1 - \alpha) \cdot R_0$$  \hspace{1cm} (3)

Where $R''$ is the average effective reproduction rate for the population; $R'$ is the effective reproduction rate for people who participate in the test program; $R_0$ is the basic reproduction rate; and $\alpha$ is the share of the population who participate in the test program.

If – following Romer – $R_0 = 2.5$, it is easy to see that $\alpha \geq 0.60$ is a necessary condition to make it theoretical possible for $R'' \leq 1$. This basically says that, if $\alpha = 0.60$, then we need $R' = 0$ to keep $R'' = 1$, i.e. if only 60 percent of the population participate in the test program, then participants need to be tested so often that they infect no-one susceptible if the virus is to be contained. This is obviously unrealistic, even in Romer’s model, since Romer operates with false negatives $(n)$. Using
Romer’s model and parameters, we can calculate $R''$ in Romer’s setup, depending on the participation rate. This is shown in Figure 1 below.

**Figure 1 Theoretical limit for $R''$ depending on participation rates and using Romer’s values**

Using Romer’s model and assumptions about $R_0$ ($=2.5$), $\tau$ ($=0.07$), $\nu$ ($=0.3$) and $I$ ($=14$), we can see from Figure 1 that at least 87 percent of the population needs to participate in the program to keep the effective reproduction rate $R'' < 1$. In the extreme case where no-one participates, $R''$ is simply equal to $R_0 = 2.5$, and in the other extreme case, where 100 percent participate, $R'' = 0.785$, which is identical to Romer’s result.

We now turn to see how this new insight about participation rates affects quarantining and test rates for participants in the test program if we want to keep $R'' \leq 1$.

The share of infected participants that needs to be quarantined can be found by inserting equation (1) into equation (3) and isolating $\varphi$ on the left hand side, which gives us equation (4):

$$\varphi = \frac{R_0 - R''}{\alpha \cdot R_0} \quad (4)$$

Now, inserting equation (2) into (4) and isolating for $t$, we get a term for the necessary fraction of the participants who need to be tested in equation (5):
Now, assume that we want to keep the effective average reproduction rate, $R''$, equal to 1. Then, the interpretation of equation (5) is straightforward. The lower the participation rate, $\alpha$, is and the higher the false negative rate, $n$, is, the more people we need to test in order to keep $R'' = 1$. Also, a larger $l$ reduces the need for testing. Here, it is worth noting that $l$ in Romer’s model can be interpreted as the number of days out of a total illness period of 14 days an infected person is isolated. Using $l = 14$, Romer implicitly assumes that that everyone who will be isolated is isolated on the first day of the illness period. This is obviously a problematic assumption and I will get back to this later.

Using Romer’s assumptions about $R_0$ (=2.5), $t$ (=0.07), $n$ (=0.3) and $l$ (= 14) and keeping $R'' = 1$, we can draw equations (4) and (5) for varying participation rates, $\alpha$, giving us Figure 2.

Figure 2 Testing and quarantining to keep $R'' = 1$ depending on participation rates

Figure 2 tells us two interesting facts. At first sight, the share of participants who need to be tested daily (the blue line) does not look like a significant problem. If only 50 percent participate in the test program, we just need to test participants twice as much as if everybody participated. However, if we look at the share of infected participants who need to be isolated, we get another picture. If only 50 percent participate in the program, 120 percent of the infected need to be isolated to keep $R'' = 1$, which is obviously impossible. The reason for this discrepancy is Romer’s implicit assumption that everyone who will be isolated is isolated on the first day of the illness period.
This assumption is clearly problematic, and in the following section, we will look at an extension to Romer’s model without this assumption.

**Extending Romer’s model: Letting length of isolation depend on how often people are tested**

In a time-continuous model inspired by (Cleevely et al. 2020), we let $R’$ be the expected number of people that a randomly chosen infected person infects before that person is positively tested (or stops being infectious, if sooner). Let $r_j$ be the expected number of individuals infected by an individual on day $i$ of his/her infection, for $j = 1, 2, ..., d$ where the length of infectivity is $d$. Thus $R’ = \sum_{j=1}^{d} r_j$. Now, suppose an individual is tested every $N = \frac{1}{t}$ days, and that $N < d$ (i.e., every person is tested at least once during his/her period of illness). If the time of the infection is random, then

$$R' = \frac{1}{1 - n} \cdot \frac{1}{N} \cdot \sum_{i=1}^{N} \sum_{j=1}^{i} r_j$$  \hspace{1cm} (6)

From this we can deduce that

$$R' = \frac{1}{1 - n} \cdot \frac{1}{N} \cdot \sum_{j=1}^{N} (N - j) \cdot r_j$$  \hspace{1cm} (7)

So, if $r_j = \frac{R_0'}{d}$ then

$$R' = R_0 \cdot \frac{1}{1 - n} \cdot n \cdot \frac{N}{2 \cdot d}$$  \hspace{1cm} (8)

And since $N = \frac{1}{t}$

$$R' = R_0 \cdot \frac{1}{1 - n} \cdot \frac{1}{2 \cdot d \cdot t}$$  \hspace{1cm} (9)

Inserting equation (9) into (3) and isolating $t$ on the left hand side, we get equation (10):

$$t = \alpha \cdot R_0 \cdot \frac{1}{1 - n} \cdot \frac{1}{2 \cdot d} \cdot \frac{1}{R'' - (1 - \alpha) \cdot R_0}$$  \hspace{1cm} (10)

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3 This is the same as 1 in Romer’s model c.f. equation (2)

4 Of course in reality such a number would need to be rounded up or down to a full number of days.

5 To keep the math simple, I have done the calculations for $t \leq 1$. However, the results hold for $t \to \infty$ where a person is tested several times every day.
Using equation (7), we can calculate the necessary share of the participants we need to test each day, if we want to keep $R''_0 = 1$. This is – using the same values as earlier – illustrated in Figure 3.

**Figure 3 Testing and quarantining to keep $R'' = 1$ in an extended model**

![Graph showing the share of participants tested each day versus participation rate.](image)

Figure 3 illustrates that the required share of participants who need to be tested each day increases dramatically and approaches infinity as the participation rate gets close to 60 percent, which is the theoretical threshold for keeping $R'' = 0$. The interpretation is clear. If exactly 60 percent of the population participate in the test program, $R' = 0$ for the participants in the test program is necessary to reach $R''_0 = 1.0$ for the population as a whole (when $R_0 = 2.5$). This means that all participants must be isolated as soon as they are infected, which means that participants must be tested “constantly”.

Figure 3 also shows that, if we want to limit the testing frequency to every 5th day (corresponding to testing 20 percent of the population every day), then at least 80 percent of the population must participate in the test program.

If 100 percent of the population participate in the test program, we need to test 13 percent of the population every day. This is almost double what Romer proposes to achieve $R' = 0.75$. If we want to bring $R''$ down to 0.75 in our model, we need to test 17 percent of the population every day. That is almost 2½ times as many people every day as Romer suggests.

**Conclusion**

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(Cleevely et al. 2020) allow for self-quarantining and a delay between performing the test and getting the result. These two effects more or less counter each other, if 25 percent self-quarantine and the delay is 1 day and in this case the results of their model is very similar to the results presented in Figure 3.
As stated earlier, it is not cost-free to participate in a test program, nor to test positive and have to isolate. However, this does not mean that it is impossible to make people participate anyway, e.g., by forcing or paying them. But 80 percent participation, for example, is a massive share of the population and getting participation rates at or around that level is likely to be enormously expensive. As a reference, only 62 percent of adult Americans voted in the 2016 presidential election, and in Denmark about 50 percent of the elderly do not get the free influenza vaccine which could potentially save their lives (Statens Serum Institut 2018). And these are rare events – not something people have to do every week. Therefore, it is likely that making people participate is a more difficult problem than making the necessary test capacity available (here, I agree with Paul Romer and encourage everybody to read Romer’s soda analogy).  

Note, however, that stating that $R'' < 1$ cannot be reached through a test program is far from saying that a test program is not a great idea. Using test and isolation as a means to reduce the effective reproduction rate is likely to be cheaper and less restrictive than mandatory lockdowns. Also, if testing and isolating only reduces $R''$, although not enough to get below 1 there are still great benefits, since herd immunity will apply at lower infection rates. Likewise, the peak number of infected is lower, when $R''$ is lower.  

Also, it may be possible to achieve $R'' < 1$ using test and isolation in combination with other measures. But it is important to note that the risk of being infected is a strong incentive for an individual to keep social distancing. Hence, as noted by (Cochrane 2020) and (Aadland, Finno, and Huang 2011), a “test and isolate”-strategy may work as a substitute rather than a compliment to voluntary social distancing, because it decreases the risk of infection for an individual not practicing distancing.  

Nevertheless, my results illustrate that we need to focus more on ideas that can encourage participation if we want a test program to be as effective as possible. The analysis also has implications for the choice of endgame since it shows that the costs of trying to avoid the spread of COVID-19 altogether while waiting for a vaccine will be higher than what the Romer analysis predicts.

7 See https://paulromer.net/if-tests-were-sodas/
References


